

# NAG C Library Function Document

## nag\_estimate\_garchGJR (g13fec)

### 1 Purpose

nag\_estimate\_garchGJR (g13fec) estimates the parameters of a univariate regression-GJR GARCH( $p, q$ ) process (see Glosten, *et al.* (1993)).

### 2 Specification

```
#include <nag.h>
#include <nagg13.h>

void nag_estimate_garchGJR (const double yt[], const double x[], Integer tdx,
    Integer num, Integer p, Integer q, Integer nreg, Integer mn,
    double theta[], double se[], double sc[], double covar[],
    Integer tdc, double *hp, double et[], double ht[], double *lgf,
    Nag_Garch_Stationary_Type stat_opt,
    Nag_Garch_Est_Initial_Type est_opt, Integer max_iter, double tol,
    NagError *fail)
```

### 3 Description

A univariate regression-GJR GARCH( $p, q$ ) process, with  $p$  coefficients  $\alpha_i$ ,  $i = 1, \dots, p$ ,  $q$  coefficients,  $\beta_i$ ,  $i = 1, \dots, q$ , mean  $b_o$ , and  $k$  linear regression coefficients  $b_i$ ,  $i = 1, \dots, k$ , can be represented by:

$$y_t = b_o + x_t^T b + \epsilon_t \quad (1)$$

$$\epsilon_t | \psi_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^q (\alpha_i + \gamma S_{t-i}) \epsilon_{t-i}^2 + \sum_{i=1}^p \beta_i h_{t-i}, \quad t = 1, \dots, T.$$

where  $S_t = 1$ , if  $\epsilon_t < 0$ , and  $S_t = 0$ , if  $\epsilon_t \geq 0$ . Here  $T$  is the number of terms in the sequence,  $y_t$  denotes the endogenous variables,  $x_t$  the exogenous variables,  $b_o$  the mean,  $b$  the regression coefficients,  $\epsilon_t$  the residuals,  $\gamma$  is the asymmetry parameter,  $h_t$  is the conditional variance, and  $\psi_t$  the information set of all information up to time  $t$ .

The routine nag\_estimate\_garchGJR provides an estimate for  $\hat{\theta}$ , the  $(p + q + k + 3) \times 1$  parameter vector  $\theta = (b_o, b^T, \omega^T)$  where  $\omega^T = (\alpha_0, \alpha_1, \dots, \alpha_q, \beta_1, \dots, \beta_p, \gamma)$  and  $b^T = (b_1, \dots, b_k)$ .

**mn**, **nreg** (see Section 4) can be used to simplify the GARCH( $p, q$ ) expression in equation (1) as follows:

#### No Regression or Mean

$$y_t = \epsilon_t,$$

$$\mathbf{mn} = 0,$$

$$\mathbf{nreg} = 0, \text{ and}$$

$$\theta \text{ is a } (p + q + 2) \times 1 \text{ vector.}$$

#### No Regression

$$y_t = b_o + \epsilon_t,$$

$$\mathbf{mn} = 1,$$

$$\mathbf{nreg} = 0, \text{ and}$$

$$\theta \text{ is a } (p + q + 3) \times 1 \text{ vector.}$$

**Note:** if the  $y_t = \mu + \epsilon_t$ , where  $\mu$  is known (not to be estimated by `nag_estimate_garchGJR`) then equation (1) can be written as  $y_t^\mu = \epsilon_t$ , where  $y_t^\mu = y_t - \mu$ . This corresponds to the case **No Regression or Mean**, with  $y_t$  replaced by  $y_t - \mu$ .

#### No Mean

$$y_t = x_t^T b + \epsilon_t,$$

$$\mathbf{mn} = 0,$$

$$\mathbf{nreg} = k \text{ and}$$

$\theta$  is a  $(p + q + k + 2) \times 1$  vector.

## 4 Parameters

**Note:** for convenience `npar` will be used here to denote the expression  $2 + \mathbf{q} + \mathbf{p} + \mathbf{mn} + \mathbf{nreg}$  representing the number of model parameters.

- 1: **yt[num]** – const double *Input*  
*On entry:* the sequence of observations,  $y_t$ ,  $t = 1, \dots, T$ .
- 2: **x[num][tdx]** – const double *Input*  
*On entry:* row  $t$  of **x** contains the time dependent exogenous vector  $x_t$ , where  $x_t^T = (x_t^1, \dots, x_t^k)$ , for  $t = 1, \dots, T$ .
- 3: **tdx** – Integer *Input*  
*On entry:* the second dimension of the array **x** as declared in the function from which `nag_estimate_garchGJR` is called.  
*Constraint:* **tdx**  $\geq$  **nreg**.
- 4: **num** – Integer *Input*  
*On entry:* the number of terms in the sequence,  $T$ .  
*Constraint:* **num**  $\geq$  **npar**.
- 5: **p** – Integer *Input*  
*On entry:* the GARCH( $p, q$ ) parameter  $p$ .  
*Constraint:* **p**  $\geq$  0.
- 6: **q** – Integer *Input*  
*On entry:* the GARCH( $p, q$ ) parameter  $q$ .  
*Constraint:* **q**  $\geq$  1.
- 7: **nreg** – Integer *Input*  
*On entry:* the number of regression coefficients,  $k$ .  
*Constraint:* **nreg**  $\geq$  0.
- 8: **mn** – Integer *Input*  
*On entry:* if **mn** = 1 then the mean term  $b_0$  will be included in the model.  
*Constraint:* **mn** = 0 or **mn** = 1.

- 9: **theta[npar]** – double *Input/Output*
- On entry:* the initial parameter estimates for the vector  $\theta$ . The first element contains the coefficient  $\alpha_o$ , the next **q** elements contain the coefficients  $\alpha_i$ ,  $i = 1, \dots, q$ . The next **p** elements are the coefficients  $\beta_j$ ,  $j = 1, \dots, p$ . The next element contains the asymmetry parameter  $\gamma$ . If **est\_opt** = **Nag\_Garch\_Est\_Initial\_False** then (when **mn** = 1) the next term contains an initial estimate of the mean term  $b_o$  and the remaining **nreg** elements are taken as initial estimates of the linear regression coefficients  $b_i$ ,  $i = 1, \dots, k$ .
- On exit:* the estimated values  $\hat{\theta}$  for the vector  $\theta$ . The first element contains the coefficient  $\alpha_o$ , the next **q** elements contain the coefficients  $\alpha_i$ ,  $i = 1, \dots, q$ . The next **p** elements are the coefficients  $\beta_j$ ,  $j = 1, \dots, p$ . The next element contains the estimate for the asymmetry parameter  $\gamma$ . If **mn** = 1 then the next element contains an estimate for the mean term  $b_o$ . The final **nreg** elements are the estimated linear regression coefficients  $b_i$ ,  $i = 1, \dots, k$ .
- 10: **se[npar]** – double *Output*
- On exit:* the standard errors for  $\hat{\theta}$ . The first element contains the standard error for  $\alpha_o$ , the next **q** elements contain the standard errors for  $\alpha_i$ ,  $i = 1, \dots, q$ , the next **p** elements are the standard errors for  $\beta_j$ ,  $j = 1, \dots, p$ . The next element contains the standard error for  $\gamma$ . If **mn** = 1 then the next element contains the standard error for  $b_o$ . The final **nreg** elements are the standard errors for  $b_j$ ,  $j = 1, \dots, k$ .
- 11: **sc[npar]** – double *Output*
- On exit:* the scores for  $\hat{\theta}$ . The first element contains the score for  $\alpha_o$ , the next **q** elements contain the score for  $\alpha_i$ ,  $i = 1, \dots, q$ , the next **p** elements are the scores for  $\beta_j$ ,  $j = 1, \dots, p$ . The next element contains the score for  $\gamma$ . If **mn** = 1 then the next element contains the score for  $b_o$ . The final **nreg** elements are the scores for  $b_j$ ,  $j = 1, \dots, k$ .
- 12: **covar[npar][tdc]** – double *Output*
- On exit:* the covariance matrix of the parameter estimates  $\hat{\theta}$ , that is the inverse of the Fisher Information Matrix.
- 13: **tdc** – Integer *Input*
- On entry:* the second dimension of the array **covar** as declared in the function from which `nag_estimate_garchGJR` is called.
- Constraint:* **tdc**  $\geq$  **npar**.
- 14: **hp** – double \* *Input/Output*
- On entry:* If **est\_opt** = **Nag\_Garch\_Est\_Initial\_False** then **hp** is the value to be used for the pre-observed conditional variance. If **est\_opt** = **Nag\_Garch\_Est\_Initial\_True** then **hp** is not referenced.
- On exit:* If **est\_opt** = **Nag\_Garch\_Est\_Initial\_True** then **hp** is the estimated value of the pre-observed conditional variance.
- 15: **et[num]** – double *Output*
- On exit:* the estimated residuals,  $\epsilon_t$ ,  $t = 1, \dots, T$ .
- 16: **ht[num]** – double *Output*
- On exit:* the estimated conditional variances,  $h_t$ ,  $t = 1, \dots, T$ .
- 17: **lgf** – double \* *Output*
- On exit:* the value of the log likelihood function at  $\hat{\theta}$ .

- 18: **stat\_opt** – Nag\_Garch\_Stationary\_Type *Input*  
*On entry:* If **stat\_opt** = **Nag\_Garch\_Stationary\_True** then Stationary conditions are enforced. If **stat\_opt** = **Nag\_Garch\_Stationary\_False** then Stationary conditions are not enforced.
- 19: **est\_opt** – Nag\_Garch\_Est\_Initial\_Type *Input*  
*On entry:* If **est\_opt** = **Nag\_Garch\_Est\_Initial\_True** then the routine provides initial parameter estimates of the regression terms ( $b_o, b^T$ ). If **est\_opt** = **Nag\_Garch\_Est\_Initial\_False** then the initial estimates of the regression parameters ( $b_o, b^T$ ) must be supplied by the user.
- 20: **max\_iter** – Integer *Input*  
*On entry:* the maximum number of iterations to be used by the optimisation routine when estimating the GARCH( $p, q$ ) parameters. If **max\_iter** is set to 0 then the standard errors, score vector and variance-covariance are calculated for the input value of  $\theta$  in **theta**; however the value of  $\theta$  is not updated.  
*Constraint:* **max\_iter**  $\geq$  0.
- 21: **tol** – double *Input*  
*On entry:* the tolerance to be used by the optimisation routine when estimating the GARCH( $p, q$ ) parameters.
- 22: **fail** – NagError \* *Input/Output*  
The NAG error parameter (see the Essential Introduction).

## 5 Error Indicators and Warnings

### NE\_BAD\_PARAM

On entry, parameter **stat\_opt** had an illegal value.  
On entry, parameter **est\_opt** had an illegal value.

### NE\_INT\_ARG\_LT

On entry, **nreg** must not be less than 0: **nreg** = *<value>*.  
On entry, **q** must not be less than 1: **q** = *<value>*.  
On entry, **p** must not be less than 0: **p** = *<value>*.  
On entry, **max\_iter** must not be less than 0: **max\_iter** = *<value>*.

### NE\_2\_INT\_ARG\_LT

On entry, **tdx** = *<value>* while **nreg** = *<value>*.  
These parameters must satisfy **tdx**  $\geq$  **nreg**.  
On entry, **tdc** = *<value>* while  $2+\mathbf{q}+\mathbf{p}+\mathbf{mn}+\mathbf{nreg}$  = *<value>*.  
These parameters must satisfy **tdc**  $\geq$   $2+\mathbf{q}+\mathbf{p}+\mathbf{mn}+\mathbf{nreg}$ .  
On entry, **num** = *<value>* while  $2+\mathbf{q}+\mathbf{p}+\mathbf{mn}+\mathbf{nreg}$  = *<value>*.  
These parameters must satisfy **num**  $\geq$   $2+\mathbf{q}+\mathbf{p}+\mathbf{mn}+\mathbf{nreg}$ .

### NE\_INVALID\_INT\_RANGE\_2

Value *<value>* given to **mn** is not valid. Correct range is 0 to 1.

### NE\_MAT\_NOT\_FULL\_RANK

Matrix  $X$  does not give a model of full rank.

**NE\_MAT\_NOT\_POS\_DEF**

Attempt to invert the second derivative matrix needed in the calculation of the covariance matrix of the parameter estimates has failed. The matrix is not positive-definite, possibly due to rounding errors.

**NE\_ALLOC\_FAIL**

Memory allocation failed.

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

**6 Further Comments****6.1 Accuracy**

Not applicable.

**6.2 References**

Engle R (1982) Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation *Econometrica* **50** 987–1008

Bollerslev T (1986) Generalised Autoregressive Conditional Heteroskedasticity *Journal of Econometrics* **31** 307–327

Engle R and Ng V (1993) Measuring and Testing the Impact of News on Volatility *Journal of Finance* **48** 1749–1777

Hamilton J (1994) *Time Series Analysis* Princeton University Press

Glosten L, Jagannathan R and Runkle D (1993) Relationship between the Expected Value and the Volatility of Nominal Excess Return on Stocks *Journal of Finance* **48** 1779–1801

**7 See Also**

None.

**8 Example**

This example program illustrates the use of `nag_estimate_garchGJR` to model a GARCH(1,1) sequence generated by `nag_generate_garchGJR` (`g05hmc`), a six step forecast is then calculated using `nag_forecast_garchGJR` (`g13ffc`).

**8.1 Program Text**

```
/* nag_estimate_garchGJR (g13fec) Example Program.
 *
 * Copyright 2000 Numerical Algorithms Group.
 *
 * NAG C Library
 *
 * Mark 6, 2000.
 */
#include <nag.h>
#include <nag_stdlib.h>
#include <stdio.h>
#include <ctype.h>
```

```

#include <math.h>
#include <nagg05.h>
#include <nagg13.h>

int main(void)
{
    double *bx=0, *covar=0, *etm=0, fac1, gamma, hp, *ht=0, *htm=0, lgf;
    double *param=0, *rvec=0, *sc=0, *se=0, *theta=0, tol;
    double mean, *x=0, xterm, *cvar=0, *yt=0;
    Integer exit_status = 0;
    Integer i, ip, iq, j, k, nt;
    Integer tdx, tdc, maxit, mn, num, num_startup, npar;
    Integer nreg, seed;
    Nag_Garch_Est_Initial_Type est_opt;
    Nag_Garch_Stationary_Type stat_opt;
    Nag_Garch_Fcall_Type fcall;
    NagError fail;

    INIT_FAIL(fail);
    num = 1000;
    mn = 1;
    mean = 4.0;
    nreg = 2;
    ip = 1;
    iq = 1;
    npar = iq + ip + 1;
    nt = 6;
    tdx = nreg;
    tdc = npar+mn+nreg+1;

#define YT(I) yt[(I)-1]
#define THETA(I) theta[(I)-1]
#define SE(I) se[(I)-1]
#define SC(I) sc[(I)-1]
#define RVEC(I) rvec[(I)-1]
#define PARAM(I) param[(I)-1]
#define HTM(I) htm[(I)-1]
#define HT(I) ht[(I)-1]
#define ETM(I) etm[(I)-1]
#define BX(I) bx[(I)-1]
#define CVAR(I) cvar[(I)-1]
#define X(I,J) x[((I)-1) * tdx + ((J)-1)]
#define COVAR(I,J) covar[((I)-1) * tdc + ((J)-1)]

    Vprintf ("g13fec Example Program Results \n\n");

    if (!(bx = NAG_ALLOC (nreg, double))
        || !(covar = NAG_ALLOC ((npar+mn+nreg+1) * (npar+mn+nreg+1), double))
        || !(etm = NAG_ALLOC (num, double))
        || !(ht = NAG_ALLOC (num, double))
        || !(htm = NAG_ALLOC (num, double))
        || !(param = NAG_ALLOC (npar+mn+nreg+1, double))
        || !(rvec = NAG_ALLOC (40, double))
        || !(sc = NAG_ALLOC (npar+mn+nreg+1, double))
        || !(se = NAG_ALLOC (npar+mn+nreg+1, double))
        || !(theta = NAG_ALLOC (npar+mn+nreg+1, double))
        || !(cvar = NAG_ALLOC (nt, double))
        || !(x = NAG_ALLOC (num*nreg, double))

```

```

    || !(yt = NAG_ALLOC (num, double)))
    {
    Vprintf("Allocation failure\n");
    exit_status = -1;
    goto END;
    }

seed = 11;
gamma = 0.1;
BX (1) = 1.5;
BX (2) = 2.5;

for (i = 1; i <= num; ++i)
    {
    fac1 = (double) i *0.01;
    X (i, 2) = sin (fac1) * 0.7 + 0.01;
    X (i, 1) = fac1 * 0.1 + 0.5;
    }

PARAM (1) = 0.4;
PARAM (2) = 0.1;
PARAM (3) = 0.7;

fcall = Nag_Garch_Fcall_True;
g05cbc(seed);
num_startup = 200;
g05hmc (num_startup, ip, iq, &PARAM (1), gamma, &HT (1), &YT (1),
        fcall, &RVEC (1), &fail);
if (fail.code != NE_NOERROR)
    {
    Vprintf("Error from g05hmc.\n%s\n", fail.message);
    exit_status = 1;
    goto END;
    }
fcall = Nag_Garch_Fcall_False;
g05hmc (num, ip, iq, &PARAM (1), gamma, &HT (1), &YT (1),
        fcall, &RVEC (1), &fail);
if (fail.code != NE_NOERROR)
    {
    Vprintf("Error from g05hmc.\n%s\n", fail.message);
    exit_status = 1;
    goto END;
    }

for (i = 1; i <= num; ++i)
    {
    xterm = 0.0;
    for (k = 1; k <= nreg; ++k)
xterm += X (i, k) * BX (k);

    if (mn == 1)
YT (i) = mean + xterm + YT (i);
    else
YT (i) = xterm + YT (i);
    }

est_opt = Nag_Garch_Est_Initial_True;
stat_opt = Nag_Garch_Stationary_True;
maxit = 50;

```

```

tol = 1e-12;

for (i = 1; i <= npar; ++i)
    THETA (i) = PARAM (i) * 0.5;
THETA (npar + 1) = gamma * 0.5;
if (mn == 1)
    THETA (npar + 2) = mean * 0.5;

for (i = 1; i <= nreg; ++i)
    THETA (npar + 1 + mn + i) = BX (i) * 0.5;

g13fec (&YT (1), &X (1, 1), tdx, num, ip, iq, nreg, mn,
        &THETA (1), &SE (1), &SC (1), &COVAR (1, 1), tdc, &hp,
        &ETM (1), &HTM (1), &lgf, stat_opt, est_opt, maxit, tol, &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from g13fec.\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

Vprintf ("          Parameter estimates          Standard errors          Correct va-
lues\n");
for (j = 1; j <= npar; ++j)
    Vprintf ("%20.4f                (%6.4f) %20.4f\n", THETA (j), SE (j),
PARAM(j));

Vprintf ("%20.4f                (%6.4f) %20.4f\n", THETA (npar+1), SE (npar+1),
gamma);
if (mn)
    Vprintf ("%20.4f                (%6.4f) %20.4f\n", THETA (npar+2), SE (npar+2),
mean);
for (j = 1; j <= nreg; ++j)
    Vprintf ("%20.4f                (%6.4f) %20.4f\n", THETA (npar+1+mn+j), SE(n-
par+1+mn+j), BX(j));

/* now forecast nt steps ahead */

gamma = THETA(npar+1);

g13ffc(num,nt,ip,iq,&THETA(1),gamma,&CVAR(1),&HTM(1),&ETM(1),&fail);

Vprintf ("\n%d step forecast = %8.4f\n",nt, CVAR(nt));

END:
if (bx) NAG_FREE (bx);
if (covar) NAG_FREE (covar);
if (etm) NAG_FREE (etm);
if (ht) NAG_FREE (ht);
if (htm) NAG_FREE (htm);
if (param) NAG_FREE (param);
if (sc) NAG_FREE (sc);
if (se) NAG_FREE (se);
if (theta) NAG_FREE (theta);
if (cvar) NAG_FREE (cvar);
if (x) NAG_FREE (x);
if (yt) NAG_FREE (yt);
if (rvec) NAG_FREE (rvec);

```

```
    return exit_status;  
}
```

## 8.2 Program Data

None.

## 8.3 Program Results

g13fec Example Program Results

Parameter estimates	Standard errors	Correct values
0.4326	(0.1356)	0.4000
0.0685	(0.0333)	0.1000
0.7173	(0.0672)	0.7000
0.1326	(0.0553)	0.1000
4.1205	(0.1730)	4.0000
1.3950	(0.1658)	1.5000
2.4518	(0.1037)	2.5000

6 step forecast = 2.2549

---